Introduction to Measurement Error: Prevalence, Impact and Adjustments

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Introduction

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  - The result of an imperfect measurement process
  - Discrepancies between the ‘true’ and the observed value

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  - The result of an imperfect measurement process
  - Discrepancies between the 'true' and the observed value
- A widespread problem, but especially prevalent in the Social Sciences
  - Elusive constructs, subjectively defined, e.g. happiness, ethnicity
  - Even when well-defined, they often are subjectively elicited: e.g. survey data, affected by memory failures (e.g. *when was the last time you went to a pub?*), social desirability (e.g. *for how long have you been unemployed*?)
  - Administrative/official data used as proxies (e.g. using earnings to measure poverty; measuring violent crime from police records)

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  - Administrative/official data used as proxies (e.g. using earnings to measure poverty; measuring violent crime from police records)
- To investigate the prevalence, impact and adjustments we need to define these errors formally using measurement error models

Defining Measurement Error Formally

- The classical measurement error model (random errors)
  - $\overbrace{X^*}^{observed} = \overbrace{X}^{true\ value} + \overbrace{U}^{noise}$
  - with the errors taken to be randomly distributed, $U \sim N(0, \sigma_U)$
  - E.g. results from an IQ test, blood pressure measurements,

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- Only the variance is affected
  - $\sigma^2_{X^*} = \sigma^2_X + \sigma^2_U$; but the mean is unaffected since $E(U) = 0$
  - Taking repeated observations we can estimate the prevalence of classical measurement error
  - The reliability ratio: $\rho_{X^*} = \dfrac{\sigma^2_X}{\sigma^2_X + \sigma^2_U} = \dfrac{\text{true variability}}{\text{observed variability}}$

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- Repeated observations won't pick up systematic errors
  - We need a *gold standard* (at least for a subgroup of our sample)
  - E.g. Unemployment register, victimisation surveys

More Errors (Multiplicative)?!

- What if the error is proportional to the true value of the quantity being measured?
  - E.g. memory failures;

    How many alcohol units do you drink per week?

    How many times have you eaten indoors (in a restaurant or similar) since 'Freedom day'?

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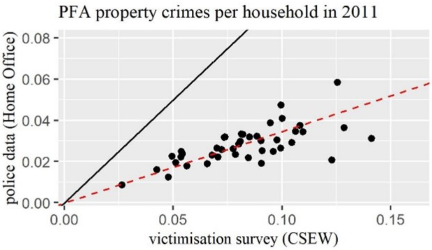
- Often we might have multiple mechanisms
  - Recounting Crime, an ESRC project where we study measurement error in police recorded crime rates
  - We have defined the measurement error as:

    i) multiplicative systematic, since not all crime is reported to the police, and the observed crime seems proportional to the true extent of crime in the area

    ii) subject to variability across areas, as a result of the different reporting rates and recording practices across police forces

Multiplicative Errors: Crime Rates



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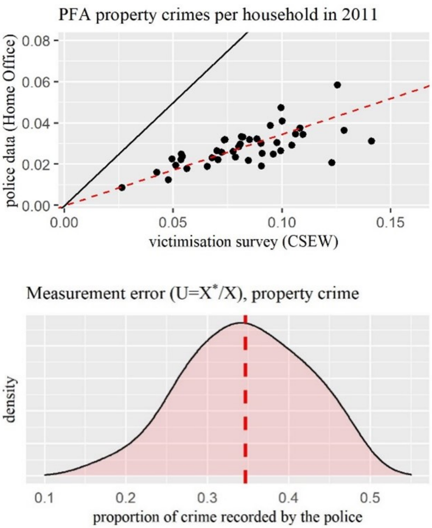

More Errors (Misclassification)?!

- Up to now we have taken examples from continuous variables
- What if the variable affected by measurement error is discrete?
  - E.g. self-reported work status, ethnicity determined through subjects' names
- We have a misclassification problem
  - which, for a binary variable, can be specified through a 2x2 misclassification matrix

$$\begin{cases} P(X^* = 1|X = 1) = \theta_{1|1}; & \text{Sensitivity} \\ P(X^* = 0|X = 0) = \theta_{0|0}; & \text{Specificity} \end{cases}$$

$$\begin{cases} P(X^* = 1|X = 0) = \theta_{1|0}; & \text{Probability false positive} \\ P(X^* = 0|X = 1) = \theta_{0|1}; & \text{Probability false negative} \end{cases}$$

The Impact of Measurement Error

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- But how does measurement error affect estimates from multivariate (regression) models?
- Assuming only one variable is prone to measurement error, its impact will depend on:
  1. the outcome model (whether linear, Poisson, etc.)
  2. the measurement error model (additive, random, etc.)
  3. where is the affected variable introduced in the model (as a response or an explanatory variable)

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   - E.g. self-reported position in a scale of xenophilia, percentage of income donated to charities
4. Systematic multiplicative errors affecting the response variable
   - $Y^* = Y \cdot U$, and $E(U) \neq 1$
   - E.g. self-reported duration of spells of unemployment, police recorded crime across areas

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    $Y \sim N(10, 3); \quad X = 15 + 3Y + \epsilon; \quad \rho_{Y,X} = 0.97$

    $\sigma_U = (0, 1, 2, 3)$

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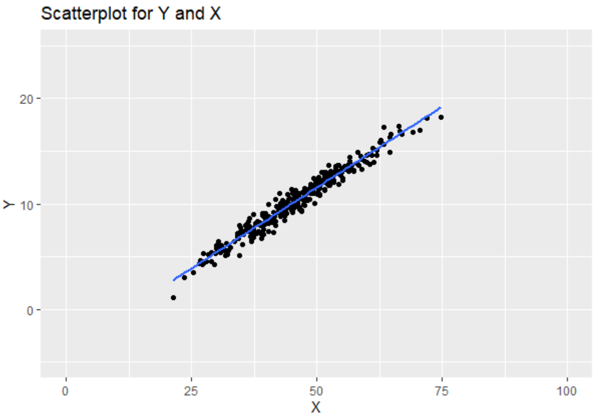

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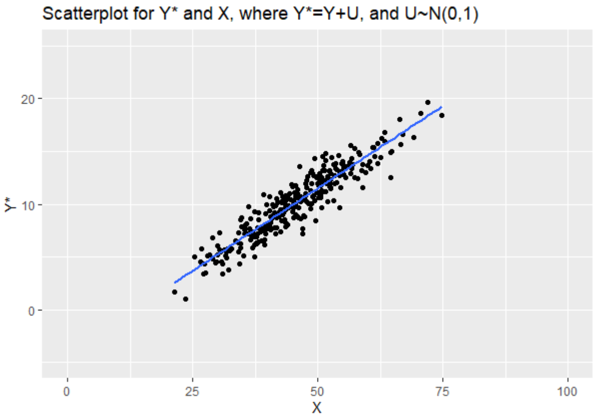

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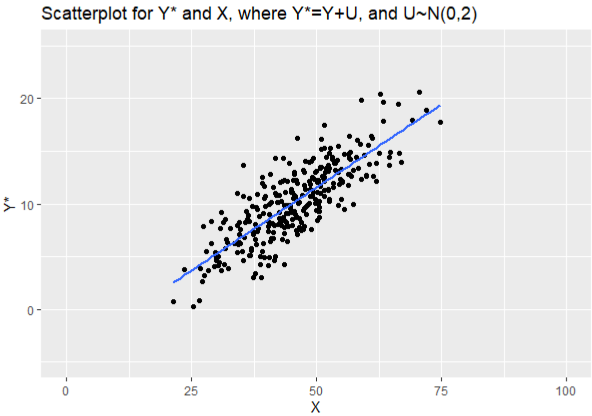

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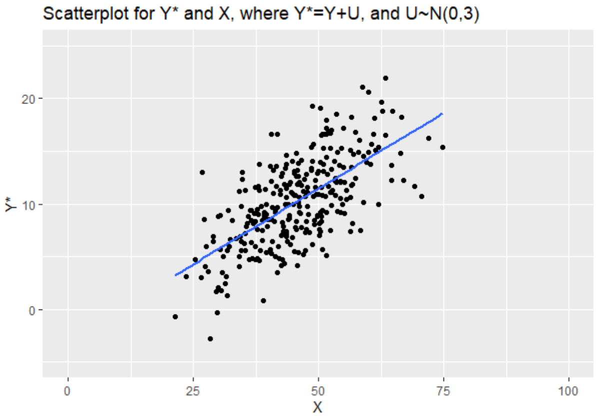

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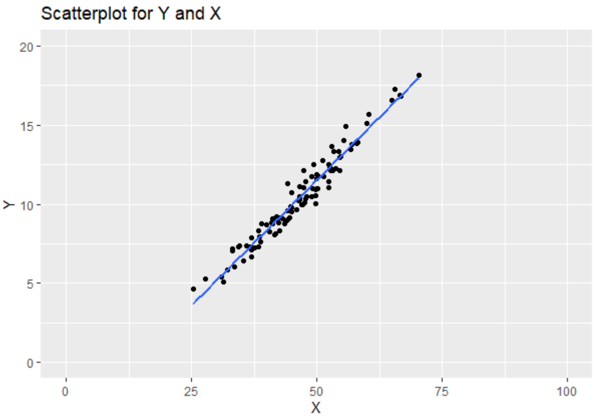

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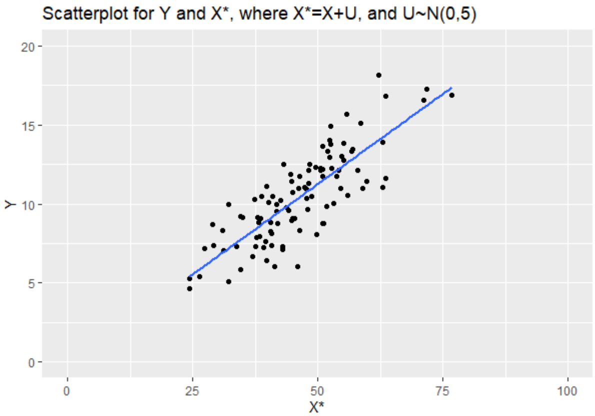

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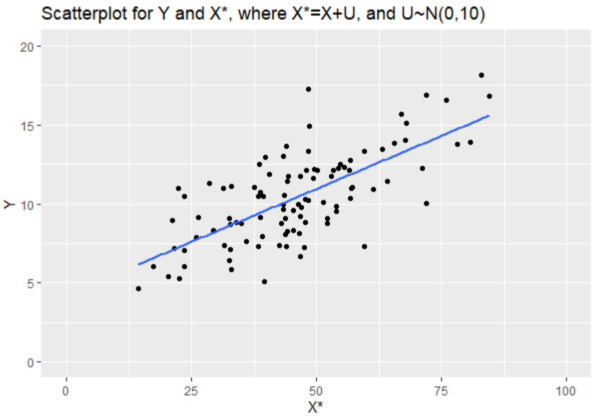

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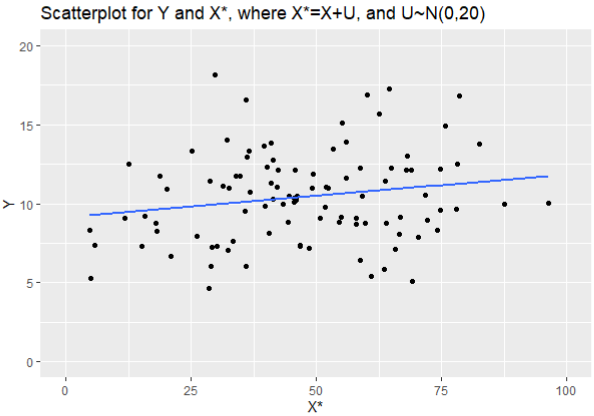

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    $Y \cdot U = \alpha + \beta X + \epsilon$

    $$Y = \frac{\alpha + \beta X + \epsilon}{U}$$
  - All regression coefficients are biased

    E.g. When modelling police crime rates ($E(U) < 1$), regression coefficients are biased upwards

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- We have only considered simple linear regression
  - When we add other explanatory variables their coefficients will also be biased, even if they are perfectly measured
  - Much harder to assess if we consider more complex models: non-linear, two-stage processes, systems of equations, etc.
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  - And we have not even consider how measurements of uncertainty are also affected
- In the words of Nugent et al. (2000: 60):
  - *"Measurement error is, to borrow a metaphor, a gremlin hiding in the details of our research that can contaminate the entire set of estimated regression parameters"*

Adjustment Methods

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  - the measurement error variability can be derived from the inter-rater reliability for a subsample of cases,
  - which can then be used to adjust the expected bias

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- Ex.2, the effect of immigration on crime rates recorded by the police (systematic multiplicative errors)
  - the under-recording can be estimated using victimisation surveys,
    $\widehat{\beta}^* = \widehat{\beta}/\bar{U}$

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  - Repeated observations (regression calibration)
- Others can be used when all you have is an educated guess (sensitivity analysis)
  - Bayesian adjustments (Gustaffson, 2003)
  - Multiple overimputation (Blackwell et al., 2017)
  - SIMEX (Cook & Stefanski, 1994)

Bayesian Adjustments

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  - Can adjust any form of measurement error
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- The hardest to implement?
  - To really exploit the full flexibility of Bayesian methods we need to use Bayesian software
  - Stan, WinBUGS

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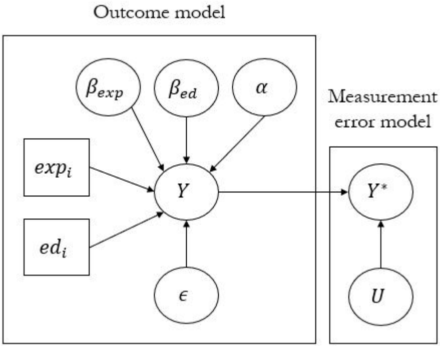

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  2. Re-estimate the outcome model using X^*_k, which produces increasingly biased estimates, $\hat{\beta}^*_k = \beta_1 \frac{S^2_X}{S^2_X + (1+\lambda_k)S^2_U}$

SIMEX

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  - Estimates the impact of measurement error through simulation
  - All we need is an understanding of the form and prevalence of the measurement error
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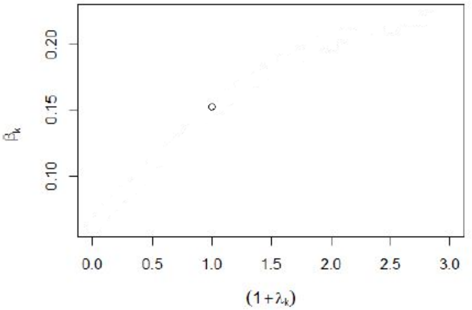
SIMEX



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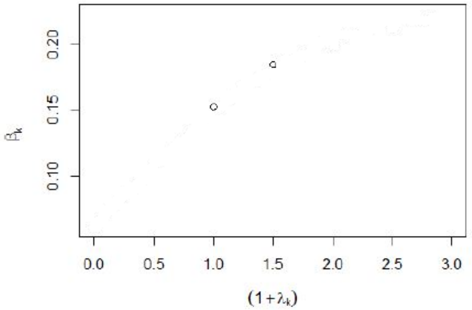

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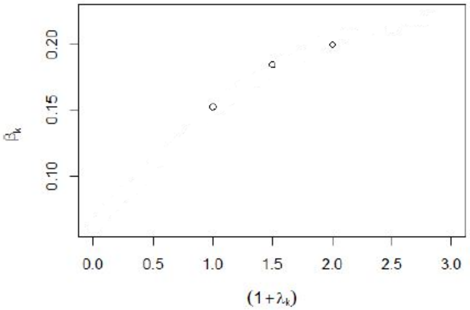

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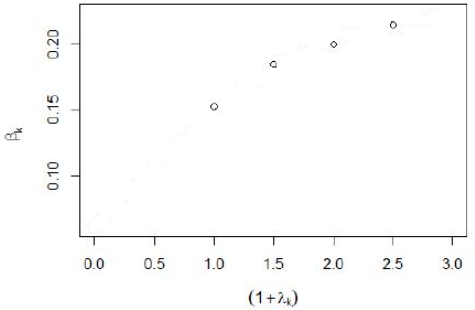

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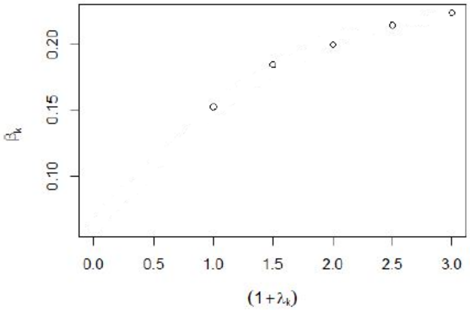

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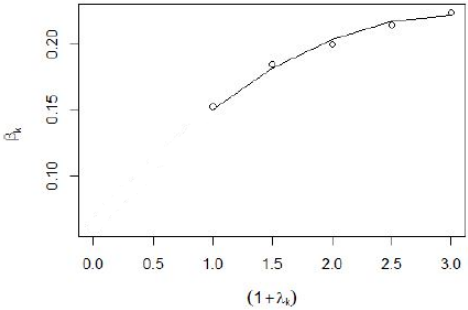

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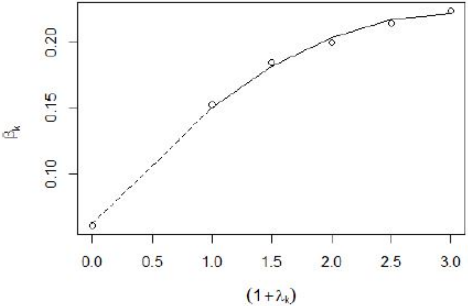

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- A much bigger problem than we tend to acknowledge
  - Affecting countless studies, potentially quite severely
- Besides improving data collection processes, we also need to employ adjustments methods
  - Many require additional data, others are quite complex
  - But there are a few methods that are simple enough, and can be used even as a sensitivity analysis tool
  - We should use them every time we suspect measurement error is present in any of our variables

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